

2.3 – Apportionment Algorithms

Wannabe Academy has 265 sophomores, 435 juniors and 350 seniors. The student council has 25 positions divided among the classes according to population.

$$\text{Ideal ratio} = \frac{\text{total population}}{\text{number of seats}}$$

The ideal ratio is ideally the number of students that each student council member will represent. (when referring to political representation, this is called the ideal district size)

$$\text{Quota} = \frac{\text{class size}}{\text{ideal ratio}}$$

Hamilton Method:

How do we round?

How do we award seats?

	Quota	Initial Apportionment	Extra Seats	Final Apportionment
Sophomores				
Juniors				
Seniors				

Jefferson Method:

How do we round?

How do we award seats?

	Quota	Initial Apportionment	Adjusted Quota d =	Final Apportionment
Sophomores				
Juniors				
Seniors				

The country of Woodland has five states. The population of each state is shown below. The House of Representatives for Woodland has 20 seats.

State	Population
Alpha	8150
Beta	5322
Gamma	3188
Delta	2353
Epsilon	987

Find the apportionment of the house seats using the Hamilton Method.

	Quota	Initial Apportionment	Extra Seats	Final Apportionment

Find the apportionment of the house seats using the Jefferson Method.

	Quota	Initial Apportionment	Adjusted Quota d =	Final Apportionment

2.4 – More Apportionment Algorithms

Wannabe Academy has 265 sophomores, 435 juniors and 350 seniors. The student council has 25 positions divided among the classes according to population.

Ideal ratio =

Webster Method

How do we round?

How do we award seats?

	Quota	Initial Apportionment	Adjusted Quota d =	Final Apportionment
Sophomores				
Juniors				
Seniors				

Hill Method

How do we round?

How do we award seats?

	Quota	Geometric Mean	Initial Apportionment	Adjusted Quota d =	Final Apportionment
Sophomores					
Juniors					
Seniors					

The country of Woodland has five states. The population of each state is shown below. The House of Representatives for Woodland has 20 seats.

State	Population
Alpha	8150
Beta	5322
Gamma	3188
Delta	2353
Epsilon	987

Find the apportionment of the house seats using the Webster Method.

	Quota	Initial Apportionment	Adjusted Quota d =	Final Apportionment

Find the apportionment of the house seats using the Hill Method.

	Quota	Geometric Mean	Initial Apportionment	Adjusted Quota d =	Final Apportionment

Apportionment Review

1. Forty seats are to be apportioned among three groups. Group A has 120 members, Group B has 198 members, and Group C has 182 members.

What is the ideal ratio? What does the ideal ratio represent in this problem?

Find the apportionment under the Hamilton method.

	Quota	Initial Apportionment	Extra Seats	Final Apportionment

Find the apportionment under the Jefferson method.

	Quota	Initial Apportionment	Adjusted Quota $d =$	Final Apportionment

Find the apportionment under the Webster method.

	Quota	Initial Apportionment	Adjusted Quota $d =$	Final Apportionment

Find the apportionment under the Hill method.

	Quota	Geometric Mean	Initial Apportionment	Adjusted Quota $d =$	Final Apportionment

2. A teacher wants to distribute 20 tokens to the local video store among three students based on the number of minutes each student spends studying, as shown the following table.

	Tom	John	Mike
Minutes Studied	825	275	50

What is the ideal ratio? What does the ideal ratio represent in this problem?

Find the apportionment under the Hamilton method.

	Quota	Initial Apportionment	Extra Seats	Final Apportionment

Find the apportionment under the Jefferson method.

	Quota	Initial Apportionment	Adjusted Quota d =	Final Apportionment

Find the apportionment under the Webster method.

	Quota	Initial Apportionment	Adjusted Quota d =	Final Apportionment

Find the apportionment under the Hill method.

	Quota	Geometric Mean	Initial Apportionment	Adjusted Quota d =	Final Apportionment

3. Four states A, B, C, and D have populations 230, 600, 4320, and 4850, respectively. They have 69 seats in the House of Representatives.

What is the ideal ratio? What does the ideal ratio represent in this problem?

Find the apportionment under the Hamilton method.

	Quota	Initial Apportionment	Extra Seats	Final Apportionment

Find the apportionment under the Jefferson method.

	Quota	Initial Apportionment	Adjusted Quota d =	Final Apportionment

Find the apportionment under the Webster method.

	Quota	Initial Apportionment	Adjusted Quota d =	Final Apportionment

Find the apportionment under the Hill method.

	Quota	Geometric Mean	Initial Apportionment	Adjusted Quota d =	Final Apportionment

4. A small country has 5 states and 240 seats in the legislature.

State	A	B	C	D	E
Population (in thousands)	427	754	4389	3873	157

What is the ideal ratio? What does the ideal ratio represent in this problem?

Find the apportionment under the Hamilton method.

	Quota	Initial Apportionment	Extra Seats	Final Apportionment

Find the apportionment under the Jefferson method.

	Quota	Initial Apportionment	Adjusted Quota d =	Final Apportionment

Find the apportionment under the Webster method.

	Quota	Initial Apportionment	Adjusted Quota d =	Final Apportionment

Find the apportionment under the Hill method.

	Quota	Geometric Mean	Initial Apportionment	Adjusted Quota d =	Final Apportionment

5. A small country consists of four states. The population of State 1 is 44,800, the population of State 2 is 52,200, the population of State 3 is 49,200, and the population of State 4 is 53,800. The total number of seats in the legislature is 100.

What is the ideal ratio? What does the ideal ratio represent in this problem?

Find the apportionment under the Hamilton method.

	Quota	Initial Apportionment	Extra Seats	Final Apportionment

Find the apportionment under the Jefferson method.

	Quota	Initial Apportionment	Adjusted Quota d =	Final Apportionment

Find the apportionment under the Webster method.

	Quota	Initial Apportionment	Adjusted Quota d =	Final Apportionment

Find the apportionment under the Hill method.

	Quota	Geometric Mean	Initial Apportionment	Adjusted Quota d =	Final Apportionment

6. A small country consists of four states. The population of State 1 is 1,251, the population of State 2 is 14,749, the population of State 3 is 5,651, and the population of State 4 is 3,349. The total number of seats in the legislature is 250.

What is the ideal ratio? What does the ideal ratio represent in this problem?

Find the apportionment under the Hamilton method.

	Quota	Initial Apportionment	Extra Seats	Final Apportionment

Find the apportionment under the Jefferson method.

	Quota	Initial Apportionment	Adjusted Quota d =	Final Apportionment

Find the apportionment under the Webster method.

	Quota	Initial Apportionment	Adjusted Quota d =	Final Apportionment

Find the apportionment under the Hill method.

	Quota	Geometric Mean	Initial Apportionment	Adjusted Quota d =	Final Apportionment

7. Consider a country consisting of three states whose populations are given in the table below. There are 50 seats in the legislature.

	A	B	C
Population (in millions)	42	75	283

What is the ideal ratio? What does the ideal ratio represent in this problem?

Find the apportionment under the Hamilton method.

	Quota	Initial Apportionment	Extra Seats	Final Apportionment

Find the apportionment under the Jefferson method.

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Find the apportionment under the Webster method.

	Quota	Initial Apportionment	Adjusted Quota d =	Final Apportionment

Find the apportionment under the Hill method.

	Quota	Geometric Mean	Initial Apportionment	Adjusted Quota d =	Final Apportionment

8. A small country has 250 seats in their legislature for six states.

	A	B	C	D	E	F
Population (in thousands)	1,646	6,936	154	2,091	685	988

What is the ideal ratio? What does the ideal ratio represent in this problem?

Find the apportionment under the Hamilton method.

[illegible]

Find the apportionment under the Jefferson method.

[illegible]

Find the apportionment under the Webster method.

[illegible]

Find the apportionment under the Hill method.

[illegible]

9. City College is made up of five different departments: communications, accounting, marketing, psychology, and technology. A total of 110 teaching positions are to be apportioned based on the school's enrollment, as shown below.

Department	Communications	Accounting	Marketing	Psychology	Technology
Enrollment	2425	745	497	230	1053

What is the ideal ratio? What does the ideal ratio represent in this problem?

Find the apportionment under the Hamilton method.

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Find the apportionment under the Jefferson method.

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Find the apportionment under the Webster method.

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Find the apportionment under the Hill method.

	Quota	Geometric Mean	Initial Apportionment	Adjusted Quota d =	Final Apportionment

10. Suppose that 68 students sign up for algebra, 59 for geometry, and 14 for precalculus. Also, there are to be six mathematics classes.

What is the ideal ratio? What does the ideal ratio represent in this problem?

Find the apportionment under the Hamilton method.

	Quota	Initial Apportionment	Extra Seats	Final Apportionment

Find the apportionment under the Jefferson method.

	Quota	Initial Apportionment	Adjusted Quota d =	Final Apportionment

Find the apportionment under the Webster method.

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Find the apportionment under the Hill method.

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11. A country has six states with populations 27,770; 25,193; 19,418; 14,612; 9217; 3790. Its House of Representatives has 40 seats.

What is the ideal ratio? What does the ideal ratio represent in this problem?

Find the apportionment under the Hamilton method.

[illegible]

Find the apportionment under the Jefferson method.

[illegible]

Find the apportionment under the Webster method.

[illegible]

Find the apportionment under the Hill method.

[illegible]

12. Mrs. Howle decided to give stickers to her children based on how many dandelions they pulled from the lawn. Rachel pulled 29 dandelions, Adam pulled 65, and Jonathan pulled 83. How could 10 stickers be awarded?

What is the ideal ratio? What does the ideal ratio represent in this problem?

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Find the apportionment under the Webster method.

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Find the apportionment under the Hill method.

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13. Apportion the 25 seats on Monroe High School's student council. The school has 385 Freshmen, 315 Sophomores, 275 Juniors, and 225 Seniors.

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